Reliability and Profit Analysis of a Metal Treatment Station (MTS) System with Effect of Waiting Time for Standby MTS

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ABSTRACT
This paper presents a Reliability model for Metal Treatment Station (MTS) system in piston foundry with facility of repair. The MTS system is used for predicting the best treatment practice based on ambient conditions melt temperature, rotor design and alloy composition. In the first model, Six Multi Level Die Block (MLDB) systems are divided into two systems containing three MLDB each and they are termed as : system one and system two. Both the systems have their own repairman. System one contains the three MLDB with one main unit i.e. Robotic A and system two also contains the three MLDB with one main unit i.e. Robotic B. Initially, two MTS appliances are operative and the third MTS is in cold standby state. For the functioning of the whole system, two MTS appliances should operate. The cold standby MTS is common for both systems. On the failure of one of the MTS, the standby MTS becomes operative and if in meanwhile, second MTS gets fail then it will wait for standby MTS system. The semi-Markov process and regenerative point technique have been used to derive the expressions for various measures of reliability. A particular case is considered to highlight the results graphically.

Keywords: MTS, MLDB systems, Robotic, semi-Markov process, regenerative point technique

INTRODUCTION
In the present scenario, the needs of modern society and the expectations of the consumers are on the rise. All consumers want maximum set-offs in their minimum earnings. Demands of the consumers catered by the producers by manufacturing goods as in many variety as possible. For the high productivity of manufacturing industry, the machines should be operated in satisfactory manner. To maintain the progress of the development, industries depend upon quality and reliability of the system used. Therefore, Reliability has become an integral part of many systems such as; hydraulic control system, computer system, electric power supply system, nuclear power plants, aircraft engine system, auto mobile engine system etc. In area of reliability researchers have been participated at great extent for development reliability models for the functioning of the systems.

Earlier researchers [1], [2], [3], [5], [8], [9], [10] and [11] have analysed the different approaches of standby systems. Many authors gave us new ideas and methods about reliability modelling used in die casting systems. Some of them are as follows: [4] dealt with analysis of stochastic modelling for

All the researchers as discussed above have dealt with reliability analysis of the die casting process in piston foundry. No information has been found on research results regarding MTS and MLDB systems in piston foundry.

Getting inspiration from the above concepts, we develop a Reliability models for Metal Treatment Station (MTS) system in piston foundry with facility of repair. The MTS system is used for predicting the best treatment practice based on ambient conditions melt temperature, rotor design and alloy composition.

In the first model, Six Multi Level Die Block (MLDB) systems are divided into two parts i.e. system one and system two. Both the systems have their own repairman. System one represents the three MLDB with main unit i.e. Robotic A and system two also represents the three MLDB with main unit i.e. Robotic B.

Initially, two MTS appliances are operative and the third MTS is in cold standby state. For the functioning of the whole system, two MTS's appliances are operative. The cold standby MTS is common for both systems. Considering the probabilities for the MTS's under the Robotic's. On the failure of any MTS, it undergoes repair by their assigned repairman. When one MTS fails, then the standby MTS becomes operative. In case, if another MTS fails, then it will be waiting for standby MTS system.

The semi-Markov processes and regenerative point technique are used to obtain following measures of system performance in steady state:

- Transition probabilities and mean sojourn times in different states.
- MTSF of the system.
- Steady state availability of the system.
- Busy period analysis of repairman.
- Expected number of visits of the repairman.
- The profit incurred to the system is evaluated and the graphical study is also made.

**MODEL DESCRIPTION AND ASSUMPTIONS:**

- The system is initially operative at state 0 as shown in Figure.1.
- The system has two states; operative state and failed state.
- All failure times are assumed to have exponential distribution whereas the repair/waiting times have general distribution.
- The repairman visits immediately as the unit fails.
- After each repair at states the system works as good as new.
- The unit is brought into operation as soon as possible.

**NOTATIONS:**

\(\lambda_1\): Failure rate of the MTS one.

\(\lambda_2\): Failure rate of MTS two.
\[ \lambda_3: \text{Failure rate of standby MTS i.e. MTS three} \]
\[ p: \text{Probability of the MTS one is under the main unit i.e. Robotic A.} \]
\[ q: \text{Probability of the MTS two is under the main unit i.e. Robotic B.} \]
\[ \beta: \text{Rate of both units under the Robotics.} \]
\[ \alpha: \text{Rate of standby MTS under waiting.} \]
\[ \gamma: \text{Repair rate of the MTS one.} \]
\[ \gamma_1: \text{Repair rate of MTS two.} \]
\[ \gamma_2: \text{Repair rate of standby MTS.} \]
\[ \gamma_3: \text{Repair rate of waiting standby MTS.} \]
\[ O_1: \text{Operative MTS one.} \]
\[ O_2: \text{Operative MTS two.} \]
\[ CS_2: \text{Cold standby MTS.} \]
\[ Rb A: \text{Main unit of the system one i.e. Robotic A.} \]
\[ Rb B: \text{Main unit of the system two i.e. Robotic B.} \]
\[ W_3: \text{Waiting for standby MTS.} \]
\[ Fr_1: \text{MTS one is under repair.} \]
\[ Fr_2: \text{MTS two is under repair.} \]
\[ Fr_3: \text{Standby MTS is under repair.} \]
\[ FR_1: \text{Repair of MTS one is continuing from previous state.} \]
\[ FR_2: \text{Repair of MTS two is continuing from previous state.} \]
\[ Fwr_1: \text{MTS one is waiting for repair.} \]
\[ Fwr_2: \text{MTS two is waiting for repair.} \]
\[ Fwr_3: \text{Standby MTS is waiting for repair.} \]
\[ F(t), f(t): \text{c.d.f. and p.d.f. of the waiting time for standby MTS.} \]
\[ G_i(t), g_i(t): \text{c.d.f. and p.d.f. of time to repair of the MTS one.} \]
\[ G_i(t), g_i(t): \text{c.d.f. and p.d.f. of time to repair of the MTS two.} \]
\[ G_i(t), g_i(t): \text{c.d.f. and p.d.f. of time to repair of the standby MTS.} \]

**TRANSITION PROBABILITIES:**

A state transition diagram showing the possible states of transition of the system is shown in Figure 1. The time period of entry into states S0, S1, S2, S3, S4, S6, S8, S9 and S11 are regenerative states. The non-zero elements pij are given below:

\[ p_{01} = p, \quad p_{02} = q, \]
\[ p_{13} = \lambda_1/(\lambda_1 + \lambda_1 \alpha), \quad p_{14} = \lambda_1 \alpha/(\lambda_1 + \lambda_1 \alpha) \]
\[ p_{28} = \lambda_2 \alpha/(\lambda_2 + \lambda_2 \alpha), \quad p_{29} = \lambda_2/(\lambda_2 + \lambda_2 \alpha) \]

**Figure 1. State Transition Diagram**
\[ p_{31} = g_1 \ast (\lambda_3), \quad p_{35} = \lambda_3 [1 - g_1 \ast (\lambda_3)] / (\lambda_3) = p_{36}^{(5)} \]
\[ p_{43} = f \ast (0), \quad p_{56} = g_1 \ast (0), \]
\[ p_{61} = g_2 \ast (\lambda_3), \quad p_{67} = \lambda_1 [1 - g_3 \ast (\lambda_3)] / (\lambda_1) = p_{63}^{(7)} \]
\[ p_{73} = g_3 \ast (0), \quad p_{89} = f \ast (0), \]
\[ p_{92} = g_2 \ast (\lambda_3), \quad p_{9,10} = \lambda_3 [1 - g_2 \ast (\lambda_3)] / (\lambda_3) = p_{9,11}^{(10)} \]
\[ p_{10,11} = g_2 \ast (0), \quad p_{12,9} = g_3 \ast (0), \]
\[ p_{11,2} = g_3 \ast (\lambda_2), \quad p_{11,12} = \lambda_2 [1 - g_3 \ast (\lambda_2)] / (\lambda_2) = p_{11,9}^{(12)} \]

By these transition probabilities it is also verified that
\[ p_{01} + p_{02} = 1, \quad p_{13} + p_{14} = 1, \quad p_{28} + p_{29} = 1, \]
\[ p_{31} + p_{35} = 1 \text{ or } p_{31} + p_{36}^{(5)} = 1, \quad p_{61} + p_{67} = 1 \text{ or } p_{61} + p_{63}^{(7)} = 1, \]
\[ p_{92} + p_{9, 10} = 1 \quad \text{or} \quad p_{92} + p_{9, 11} = 1, \quad p_{11, 2} + p_{11, 12} = 1 \quad \text{or} \quad p_{11, 2} + p_{11, 9} = 1, \]
\[ p_{63} = p_{56} = p_{73} = p_{98} = p_{10, 11} = p_{12, 9} = 1. \]

The unconditional mean time taken by the system to transit for any regenerative state ‘j’ when it (time) is counted from the epoch of entrance into state ‘i’ is mathematically stated as:

\[ m_{ij} = \int_{0}^{\infty} t \, dQ_{ij}(t) = -q_{ij}'(0) \]

\[ m_{01} + m_{02} = \frac{1}{\beta} = \mu_0, \quad m_{13} + m_{14} = \mu_1, \quad m_{28} + m_{29} = \mu_2 \]
\[ m_{40} = \mu_4, \quad m_{56} = \mu_5, \quad m_{73} = \mu_7, \quad m_{89} = \mu_8, \quad m_{10,11} = \mu_{10}, \quad m_{12,9} = \mu_{12}. \]

MEAN TIME TO SYSTEM FAILURE:

Mean time to system failure (MTSF) of the system is determined by considering failed state as absorbing state when system starts from initial state \( S_0 \) is

\[ \text{MTSF} = T_0 = \frac{1 - \phi_0^*(s)}{s} \quad (1) \]

Using L’Hospital Rule & putting the value of \( \phi_0^*(s) \), we have

\[ T_0 = \frac{N}{D} \]

where

\[ N = \mu_0 [1 - p_{13}p_{31} - p_{14}p_{41} + p_{13}p_{14}p_{29}p_{92}] + \mu_1 [p_{92} - p_{92}p_{13}p_{31}] + \mu_2 [p_{92} - p_{92}p_{13}p_{31}] + \mu_3 [p_{92} + p_{92}p_{13}p_{31} - p_{92}p_{13} + p_{92}p_{13}p_{31}] \]
\[ \& \]
\[ D = [1 - p_{13}p_{31} + p_{92} + p_{92}p_{13}p_{31}] \]

AVAILABILITY ANALYSIS (SYSTEM 1):

Using the theory of regenerative processes, the availability \( A_0 \) of the system is given by

\[ A_0 = (sA_0^*(s)) = \frac{N_1}{D_1} \quad (2) \]

where

\[ N_1 = M_1 [p_{92} (1 - p_{36}) p_{93} (7)] + M_3 [p_{92}] + M_6 [p_{92} p_{36} (5)] \]
\[ D_1 = \mu_1 [p_{92} + p_{92}p_{36} (5)] + \mu_2 + \mu_6 [p_{36} (5)] - m_4 [p_{92}p_{14} + p_{92}p_{14}p_{36} (5)] \]
AVAILABILITY ANALYSIS (SYSTEM 2):

Using the theory of regenerative processes, the availability $A_{01}$ of the system is given by

$$A_{01} = \left(sA'_{01}(s)\right) = \frac{N_{2}(2)}{D_{2}}$$  \hspace{1cm} (5)

where

$$N_{2}(2) = M_{2}[1 - p_{28}p_{02} - p_{01,11}(10)] + M_{2}[p_{02}(1 - p_{11,9}(12)p_{9,11}(10))] + M_{2}[p_{02} + M_{11}[p_{02}p_{9,11}(10)]] \hspace{1cm} (6)$$

$$D_{2} = \mu_{2}[p_{02} + p_{11,29,11}(10)] + \mu_{9}[p_{11,2}] + \mu_{11}[p_{91,1}(10)] - m_{09} [p_{28}p_{29} + p_{28}p_{11,29,11}(10)] \hspace{1cm} (7)$$

BUSY PERIOD ANALYSIS OF A REPAIRMAN FOR SYSTEM 1:

Busy period analysis of a repairman is given by

$$B_{01} = \frac{N_{2}(1)}{D_{1}}$$  \hspace{1cm} (8)

where

$$N_{2}(1) = W_3[p_{01}] + W_4[p_{01}p_{14}(1 - p_{36}(5)p_{03}(7))] + W_6[p_{01}p_{36}(5)] \hspace{1cm} (9)$$

& $D_1$ is already specified in equation (4).

BUSY PERIOD ANALYSIS OF A REPAIRMAN FOR SYSTEM 2:

Busy period analysis of a repairman is given by

$$B_{02} = \frac{N_{2}(2)}{D_{2}}$$  \hspace{1cm} (10)

where

$$N_{2}(2) = W_8[p_{02}p_{28}(1 - p_{11,9}(12)p_{9,11}(10))] + W_9[p_{02}] + W_{11}[p_{28}p_{9,11}(10)] \hspace{1cm} (11)$$

& $D_2$ is already specified in equation (7).

EXPECTED NUMBER OF VISITS OF REPAIRMAN FOR SYSTEM 1:

Expected no. of visits of repairman is given by

$$V_{01} = \frac{N_{3}(1)}{D_{1}}$$  \hspace{1cm} (12)
where

\[ N_{3(1)} = [p_{01} (1 - p_{34}^{(5)} p_{01}^{(7)})] \]  \hspace{1cm} (13)

& \text{D}_1 \text{ is already specified in equation (4).}

**EXPECTED NUMBER OF VISITS OF REPAIRMAN FOR SYSTEM 2:**

Expected no. of visits of repairman is given by

\[ V_{02} = \frac{N_{3(2)}}{D_{2}} \]  \hspace{1cm} (14)

where

\[ N_{3(2)} = [p_{02} (1 - p_{11, 9}^{(12)} p_{9, 11}^{(10)})] \]  \hspace{1cm} (15)

& \text{D}_2 \text{ is already specified in equation (7).}

**PROFIT ANALYSIS:**

The expected total profit acquired to the system is given by

\[ P = C_0 A_0 + C_1 A_{01} - C_2 B_{01} - C_3 B_{02} - C_4 V_{01} - C_5 V_{02} \]  \hspace{1cm} (16)

where

- \( C_0 \) = Revenue per unit up time of the system one.
- \( C_1 \) = Revenue per unit up time of the system two.
- \( C_2 \) = Cost per unit up-time for system one, when the repairman is busy.
- \( C_3 \) = Cost per unit up-time for system two, when the repairman is busy.
- \( C_4 \) = Cost per visit of a repairman for system one.
- \( C_5 \) = Cost per visit of a repairman for system two.

**PARTICULAR CASES:**

The following particular cases are considered for graphical representation. Let us suppose that:--

\[ f(t) = \alpha e^{-\alpha t}, \quad g_1(t) = \alpha_1 e^{-\alpha_1 t}, \quad g_2(t) = \alpha_2 e^{-\alpha_2 t}, \quad g_2(t) = \alpha_2 e^{-\alpha_2 t} \]

*Therefore, we Have*

- \( p_{01} = p \)
- \( p_{02} = q \)
- \( p_{13} = \lambda_1 / (\lambda_1 + \lambda_1 \alpha) \)
- \( p_{14} = \lambda_1 \alpha / (\lambda_1 + \lambda_1 \alpha) \)
- \( p_{28} = \lambda_2 \alpha / (\lambda_2 + \lambda_2 \alpha) \)
- \( p_{29} = \lambda_2 / (\lambda_2 + \lambda_2 \alpha) \)
\[ p_{31} = \alpha_1 / (\lambda_3 + \alpha_1) = p_{44} \]
\[ p_{41} = \alpha_2 / (\lambda_2 + \lambda_1 + \alpha_2) \]
\[ p_{49} = \lambda_1 / (\lambda_2 + \lambda_1 + \alpha_2) = p_{43} \]
\[ \mu_0 = 1/\beta = K_0 \]
\[ \mu_1 = 1 / (\lambda_1 + \lambda_2 + \alpha) \]
\[ \mu_2 = 1 / (\lambda_2 + \lambda_2 + \alpha) \]
\[ \mu_4 = 1 / (\lambda_1 + \alpha) \]
\[ \mu_{11} = 1 / (\lambda_1 + \alpha_3) \]
\[ \mu_4 = \mu_5 = \mu_7 = \mu_8 = 1 / \alpha \]

\[ p_{32} = \lambda / (\lambda_1 + \lambda_2 + \alpha_1), \]
\[ p_{48} = \lambda / (\lambda + \lambda_1 + \alpha_2), \]
\[ p_{20} = p_{51} = p_{93} = p_{73} = p_{84} = p_{14} = 1 \]

GRAPHICAL REPRESENTATION AND CONCLUSION:

On the basis of gathered information, i.e.,

(Table 1.1)

<table>
<thead>
<tr>
<th>Failure Rate of first MTS</th>
<th>( \lambda _1 )</th>
<th>0.0013853/hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Failure Rate of second MTS</td>
<td>( \lambda _2 )</td>
<td>0.00138351/hr</td>
</tr>
<tr>
<td>Failure Rate of standby MTS</td>
<td>( \lambda _3 )</td>
<td>0.001378571/hr</td>
</tr>
<tr>
<td>Repair Rate of first MTS</td>
<td>( \alpha _1 )</td>
<td>0.010262569/hr</td>
</tr>
<tr>
<td>Repair Rate of second MTS</td>
<td>( \alpha _2 )</td>
<td>0.01158713/hr</td>
</tr>
<tr>
<td>Repair Rate of standby MTS</td>
<td>( \alpha _3 )</td>
<td>0.018391145/hr</td>
</tr>
</tbody>
</table>

And the rest of values are taken as assumed and are given in Table 1.2:

(Table 1.2)

<table>
<thead>
<tr>
<th>Rate of Metal Treatment</th>
<th>( \beta )</th>
<th>1.1762493/hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of standby unit to become available</td>
<td>( \alpha )</td>
<td>2.389E-05/hr</td>
</tr>
<tr>
<td>Probability that raw material is available for system one</td>
<td>( p )</td>
<td>0.75</td>
</tr>
<tr>
<td>Probability that raw material is available for system two</td>
<td>( q )</td>
<td>0.25</td>
</tr>
<tr>
<td>Revenue per unit up time of the system one</td>
<td>( C_0 )</td>
<td>Rs.452784</td>
</tr>
<tr>
<td>Revenue per unit up time of the system two</td>
<td>( C_1 )</td>
<td>Rs.450990</td>
</tr>
<tr>
<td>Cost per unit up time during which the repairman is busy in repair for system one</td>
<td>( C_2 )</td>
<td>Rs.2545</td>
</tr>
</tbody>
</table>
Various measures of system effectiveness are evaluated in the Table 1.3 as below:

(Table1.3)

<table>
<thead>
<tr>
<th>Metric</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Time to System Failure</td>
<td>$T_0$</td>
<td>$6815.085938$ hr</td>
</tr>
<tr>
<td>Availability</td>
<td>$A_0$</td>
<td>$1.017106$</td>
</tr>
<tr>
<td>Busy Period of Repairman</td>
<td>$BR_0$</td>
<td>$0.111112$</td>
</tr>
<tr>
<td>Expected No. of Visits</td>
<td>$VR_0$</td>
<td>$0.01383$</td>
</tr>
<tr>
<td>Profit</td>
<td>$P$</td>
<td>$Rs.455132.8594$</td>
</tr>
</tbody>
</table>

Graphical study has been made for the MTSF, Profit with respect to failure rate of MTS one, revenue per unit uptime of the system (C0) for different values of cost of repairman for busy in doing repair (C2) and (C3) for system one and system two respectively. 

As shown in figure 2. The behaviour of MTSF w.r.t. rate of failure of MTS one ($\lambda_1$) for the different values of the rate of failure of MTS two ($\lambda_2$). It clear from the figure that MTSF gets decreased with increase in values of rate of failure of MTS one ($\lambda_1$). Also MTSF decreases as failure rate of MTS two ($\lambda_2$) increases.

As shown in figure 3. Nature of profit w.r.t to rate of failure of MTS one ($\lambda_1$) for the different values of rate of failure of MTS two ($\lambda_2$). As the failure rate of MTS one ($\lambda_1$) increases, the profit of the system decreases. Also, on the increase in the failure rate of MTS two ($\lambda_2$), profit decreases.

Figure 2. MTSF v/s Failure Rate
MTSF V/S RATE OF FAILURE OF FIRST MTS($\lambda_1$) FOR DIFFERENT VALUES OF RATE OF FAILURE OF SECOND MTS($\lambda_2$)

$\alpha=.00002389401, \beta=1.1762493, \alpha_1=.010262569,$
$\alpha_2=.01158713, \alpha_3=.018391145, p=.75, q=.25$

PROFIT V/S RATE OF FAILURE IN THE FIRST MTS($\lambda_1$) FOR DIFFERENT VALUES OF RATE OF FAILURE IN THE SECOND MTS($\lambda_2$)

$\alpha=.00002389401, \beta=1.1762493, \delta_1=0.1357789, \delta_2=0.19357867,$
$\delta_3=0.15679909, \delta_4=0.20789548, \alpha_1=0.10262569, \alpha_2=0.1158713,$
$\alpha_3=0.18391145, p_1=.75, p_2=.75, q_1=.25,$
$\lambda_2 = 0.001385351$
$\lambda_2 = 0.000139708$
$\lambda_2 = 0.000083871$
Figure 3. Profit v/s Failure Rate

Figure 7. Depicts the behaviour of the profit w.r.t. revenue per unit uptime of the system (C₀) for different values of cost of repairman is busy under repair for system one (C₂). The graph exhibits that there is inclination in the trend of profit increases with increases in the values of C₀. Also, following conclusions can be drawn from the above graph:

1. For C₂ = 2545, the profit is positive or zero or negative according as C₀ > or = or < 375. Hence, for this case the revenue per unit up time should be fixed greater than 375.
2. For C₂ = 19545, the profit is positive or zero or negative according as C₀ > or = or < 1345. Hence, for this case the revenue per unit up time should be fixed greater than 1345.
3. For C₂ = 35545, the profit is positive or zero or negative according as C₀ > or = or < 2386. Hence, for this case the revenue per unit up time should be fixed greater than 2386.

Figure 8. Also, depicts the behaviour of the profit w.r.t. revenue per unit uptime of the system (C₀) for different values of cost of repairman is busy under repair for system two (C₃). The graph exhibits that there is inclination in the trend of profit increases with increases in the values of C₀. Also, following conclusions can be drawn from the above graph:

1. For C₃ = 1498, the profit is positive or zero or negative according as C₀ > or = or < 465. Hence, for this case the revenue per unit up time should be fixed greater than 465.
2. For C₃ = 15498, the profit is positive or zero or negative according as C₀ > or = or < 1485. Hence, for this case the revenue per unit up time should be fixed greater than 1485.
3. For C₃ = 29998, the profit is positive or zero or negative according as C₀ > or = or < 2000. Hence, for this case the revenue per unit up time should be fixed greater than 2000.
CONCLUSION:
This work concludes the importance of the application of reliability in piston foundry. MTS system unfortunately can contain defects, which may render them unsuitable for service, resulting in higher costs and/or lower profits for production foundry. Results from the mathematical measurements and graphs that MTSF and Profit gets decreased with increase in the value of failure rates are must be used to better understanding the essential real influencing factors and to subsequently enhance the reliability model. It also asserts that the results of this research will be useful in MTS system problems.

REFERENCES:


